Eur. Phys. J. D **11**, 301–307 (2000)

THE EUROPEAN PHYSICAL JOURNAL D
EDP Sciences

Modulational interactions in magnetised piezoelectric semiconductor plasmas

G. Sharma[a] and S. Ghosh[b]

School of Studies in Physics, Vikram University, Ujjain-456 010, Madhya Pradesh, India

Received 12 August 1999 and Received in final form 17 December 1999

Abstract. Based on hydrodynamic model of plasmas an analytical investigation of frequency modulational interaction between copropagating high frequency pump and acoustic mode and consequent amplification (steady-state and transient) of the modulated waves is carried out in a magnetised piezoelectric semiconductor medium. The phenomenon of modulational amplification is treated as four wave interaction process involving cubic nonlinearity of the medium. Gain constants, threshold-pump intensities and optimum-pulse duration for the onset of modulational instabilities are estimated. The analysis has been performed in non-dispersive regime of the acoustic mode, which is one of the preconditions for achieving an appreciable initial steady-state growth of the modulated signal wave. It is found that the transient gain constant diminishes very rapidly if one chooses the pump pulse duration beyond the maximum gain point. Moreover, the desired value of the gain can be obtained by adjusting intensity and pulse duration of the pump and doping concentration of the medium concerned.

PACS. 52.35.Mw Nonlinear waves and nonlinear wave propagation (including parametric effects, mode coupling, ponderative effects, etc.) – 42.65.Ky Harmonic generation, frequency conversion – 42.50.Md Optical transient phenomena: quantum beats, photon echo, free-induction decay, dephasings and revivals, optical nutation, and self-induced transparency

1 Introduction

The problems of interaction of high-power incident radiation with plasmas are of outstanding interest in plasma physics [1,2]. High-power lasers and strong radiofrequency sources are being developed or planned for future use in order to obtain fusion energy. The possibility of obtaining fusion energy depends to a large extent on the success of technological developments of such high power devices, and to an equally high degree on a full understanding of the basic problem of how electromagnetic energy of these intense radiation fields may couple or may be forced to couple most efficiently in the fusion plasma or laboratory plasma or solid state plasma etc. At high power levels that are nowadays not only attracting the attention of the fusion plasma physicists but also that of the basic physicists dealing with nonlinear interactions.

The breakdown of superposition principle in a nonlinear medium leads to interaction between waves of different frequencies. There exists a number of nonlinear interactions which can be classified as modulational interaction of coupled modes. By modulational interactions of coupled modes and consequent amplification of decay channels, one generally refers to as an instability of wave propagating in nonlinear dispersive medium such that the steady-state becomes unstable and evolves into a temporally modulated state [3]. The concept of transverse modulational instability originates from a space-time analogy that exists when the dispersion is replaced by diffraction [4]. The well-known instability of a plane wave in a self-focusing Kerr-medium [5] is an example of transverse modulation instability.

Modulational interaction of propagating beams has been a field of interest since the origin of physical optics with its concentration on diffraction and wave guiding processes. This is due to the fact that the scattering of light from sound or low frequency electromagnetic wave affords a convenient means of controlling the frequency, intensity and direction of an optical beam [6]. The type of modulation makes possible a large number of applications involving the transmission, display and processing of information [7]. The fabrication of some optical and microwave devices such as acousto-optic modulators and low noise amplifiers are based on the interaction of an acoustic wave (AW) or a low-frequency electromagnetic wave with the incident pump beam [8,9].

The modulation problems have been studied theoretically by a number of workers [10,11] due to its vast technological potentialities. An important field of study in nonlinear acoustics is amplification/attenuation and frequency mixing of waves in semiconductors (specially III-V semiconductors) [12–14] because of its immediate relevance to problems of optical communication systems.

[a] *Permanent address*: S. R. J. Govt. Girls' College, Neemuch-458 441 (M.P.), India.
[b] e-mail: drsanjayghosh@hotmail.com

The importance of semiconductor crystals lies largely in the presence of free-carrier states and the photo generation of carriers. Looking at the potential of the semiconductors in modern optoelectronic technology, the analytical investigations of some basic nonlinear processes in such crystals are of considerable significance. These studies become more important when the media concerned are piezoelectric semiconductors mainly because of the consideration of energy gain or loss.

A large amount of work on modulational instability in solid-state plasmas is available in references [15–19]. Anderson *et al.* [20] have found that growth rate of instability in $LiNbO_3$ is large enough for an experimental demonstration of amplitude modulation and envelop soliton. The most recent activity in this area has been due to the modes of wave propagation in optical fibres [7,21,22].

The discussion have so for been restricted to steady-state type of solutions [8,11,18]. The deformation in time of laser pulse propagating in a medium with an intensity dependent index of refraction provides an example of transient nonlinear effect [23]. Such transient solutions *i.e.* combined growth of monochromatic waves in space and time are of practical importance because the powerful laser pump is often a short travelling pulse.

In centrosymmetric semiconductors the dominant nonlinear optical processes may be described in terms of electric polarisation which is a cubic function of electric field amplitude. The third-order nonlinear optical susceptibility $\chi^{(3)}$ is in general a complex quantity and is capable of describing the interference between various resonant and nonresonant processes [24]. The third-order susceptibility tensor $\chi^{(3)}$ can be conveniently used to explain modulation processes in a Kerr active medium.

In the light of above, in the present paper, by considering that the origin of modulational interaction lies in the third-order optical susceptibility $\chi^{(3)}$ arising from the nonlinear induced current density; an analytical investigation of modulational interaction between copropagating pump beam and internally generated acoustic mode (due to piezoelectricity) is presented in magnetised piezoelectric semiconductor plasma medium; as a result of which AW is amplified at the expense of pump. With the help of coupled-mode theory of plasmas, we have studied steady-state and transient amplification characteristic of modulated waves. However, as far as we know, no such attempt has been made to determine transient growth rate *via* steady-state growth rate of the modulated wave in magnetised semiconductor plasmas. Finally exhaustive numerical analysis have been performed with a set of data appropriate for a piezoelectric crystal duly irradiated by a frequency doubled and pulsed 10.6 μm CO_2 laser to establish the validity of this model.

2 Theoretical formulation

This section is devoted to theoretical formulation in which we have considered the well-known hydrodynamic model of a homogeneous semiconductor plasma of infinite extent to study the modulational interaction between intense pump wave and acoustic signal and consequent amplification (steady-state and transient) of the modulated wave, in an obliquely magnetised *n*-type piezoelectric semiconductor. A semiconductor plasma medium is found to offer the greatest device potential. We know that if the unit cell of a crystal contains at least two different atoms, the crystal can exhibit piezoelectricity. In such materials, a part of mechanical energy of the vibrations is in the form of electrical energy; the AW is then accompanied by an electromagnetic wave. Hence one may expect strong coupling between AW and the electromagnetic wave in piezoelectric semiconductors. It is assumed that the origin of the said interaction lies in the effective third-order nonlinear optical susceptibility $\chi_{\rm e}^{(3)}$. It is obvious from a simple symmetry argument that $\chi^{(2)}$ and higher even order terms are zero if the medium possesses a centre of symmetry. Nonlinear phenomena in these centrosymmetric crystals (such as cubic crystals) thus depend on $\chi^{(3)}$ and higher odd order terms. In such materials, four wave mixing processes can be induced at comparatively lower magnitude of pump field $\mathbf{E}_0$. Hence the medium is considered to be a an *n*-type centrosymmetric or nearly centrosymmetric and immersed in static magnetic field $\mathbf{B}_{\rm s}$ arbitrarily lying in *x*-*z* plane and inclined at an angle θ with *x*-direction. The medium is irradiated by an intense hybrid but plane pump wave expressed using plane wave approximation as

$$\mathbf{E}_0 = (E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}}) \exp\left[\mathrm{i}(k_0 x - \omega_0 t)\right]. \quad (1)$$

Many of the reported cases correspond to the propagation of a pump wave exactly parallel to the external magnetic field [25]. Such an exact parallel propagation can not be feasible experimentally. On the other hand earlier workers considered the electric field of the pump parallel to the propagation vector [12]. This again is not a realistic situation. For a finite semiconductor plasma, $\mathbf{E}_0$ must have components both parallel and perpendicular to the propagation vector. Thus here we have considered a hybrid pump wave. Here the use of hydrodynamic model enables us to replace the electron plasma or the streaming electrons by a charged fluid characterised by a few macroscopic parameters like mean carrier density, mean velocity etc. of the plasma fluid and thus makes the analysis of modulational interaction and other related phenomena, simple. The basic equations governing the modulational instability are

$$\frac{\partial \mathbf{v}_0}{\partial t} + \nu \mathbf{v}_0 = \frac{e}{m}\left[\mathbf{E}_0 + \mathbf{v}_0 \times \mathbf{B}_{\rm s} + \mathbf{v}_0 \times \mathbf{B}_0\right], \quad (2)$$

$$\frac{\partial \mathbf{v}_1}{\partial t} + \mathbf{v}_0 \frac{\partial \mathbf{v}_1}{\partial x} + \nu \mathbf{v}_1 = \frac{e}{m}\left[\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_{\rm s}\right], \quad (3)$$

$$\frac{\partial n_1}{\partial t} + n_{\rm e} \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0, \quad (4)$$

$$\frac{\partial E_a}{\partial x} + \frac{\beta}{\varepsilon} \frac{\partial^2 u}{\partial x^2} = \frac{n_1 e}{\varepsilon}, \quad (5)$$

$$\frac{\partial^2 u}{\partial t^2} + 2\gamma_a \frac{\partial u}{\partial t} + \frac{\beta}{\rho} \frac{\partial E_a}{\partial x} = \frac{c}{\rho} \frac{\partial^2 u}{\partial x^2} \cdot \quad (6)$$

Equations (2, 3) are the linearised momentum transfer equations under the influence of pump and signal waves.

Here $\mathbf{v}_0$ and $\mathbf{v}_1$ are the zeroth and first order oscillatory fluid velocities and $\mathbf{B}_\mathrm{s}$ and $\mathbf{B}_0$ are the applied static and pump magnetic fields, respectively. ν is the phenomenological collision frequency of electrons. We have neglected the term $\mathbf{v}_0\,(\partial \mathbf{v}_0/\partial x)$ in equation (2) under the assumption $\omega_0 \gg \nu \gg \mathbf{k}_0 \cdot \mathbf{v}_0$. In equation (3) the Lorentz force due to the terms $\mathbf{v}_0 \times \mathbf{B}_1$ and $\mathbf{v}_1 \times \mathbf{B}_0$ can safely be neglected if we assume that the transverse acoustic wave is propagating in such a direction of the crystal that it produces a longitudinal electric field *viz.* in n-InSb if $\mathbf{k}$ is along (011) and the lattice displacement $\mathbf{u}$ is along (100) so that the electric field induced by the wave is longitudinal. Equation (4) is the continuity equation with n_e and n_1 being the unperturbed and perturbed electron densities respectively. The Poisson equation (Eq. (5)) determines the space charge field $\mathbf{E}_a$. ε and β are the total permittivity and piezoelectric constant of the medium, respectively. ε is equivalent to $\varepsilon_0\varepsilon_1$; ε_0 and ε_1 being free space and high frequency dielectric constants of the crystal, respectively. Equation (6) describes lattice vibrations in the piezoelectric crystal of material density ρ. γ_a and c are the respective damping parameter and the crystal elastic constant. $\mathbf{u}(x,t) = \mathbf{u}\exp\left[\mathrm{i}(k_a x - \omega_a t)\right]$ denotes displacement of lattice points from their mean position. It is assumed that the acoustic wave frequency is much lower than the pump wave frequency $\omega_a \ll \omega_0$.

Physically a signal wave creates acoustic perturbation $(\omega_a, \mathbf{k}_a)$ and consequent electron density perturbation at the acoustic frequency, which couples nonlinearly with the incident pump wave to generate a modulated wave at the frequencies $\omega_0 \pm \omega_a$. Following Guha *et al.* [26] and using above mentioned equations in the collision dominated regime $(\nu \gg \mathbf{k}_0 \cdot \mathbf{v}_0,\ \omega_a)$, we get

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + \overline{\omega}_\mathrm{p}^2 n_1 - \frac{n_\mathrm{e} e\beta}{m\varepsilon}\frac{\partial^2 u}{\partial x^2}\frac{(\omega_{\mathrm{c}x}^2+\nu^2)}{(\omega_\mathrm{c}^2+\nu^2)} = \overline{E}_0\frac{\partial n_1}{\partial x} \tag{7}$$

where

$$\overline{E}_0 = \frac{e}{m}\left|\mathbf{E}_0 + \mathbf{v}_0 \times \mathbf{B}_\mathrm{s}\right|, \quad \overline{\omega}_\mathrm{p}^2 = \omega_\mathrm{p}^2\frac{(\omega_{\mathrm{c}x}^2+\nu^2)}{(\omega_\mathrm{c}^2+\nu^2)}$$

with $\omega_{\mathrm{c}x,z}$ $(= -eB_{\mathrm{s}x,z}/m)$ are the x and z components of cyclotron frequency and ω_p $(= \sqrt{n_\mathrm{e}e^2/m\varepsilon})$ is the electron plasma frequency of the medium. The Doppler shift has been neglected under the assumption $\omega_0 \gg \nu \gg \mathbf{k}_0\cdot\mathbf{v}_0$. In the above analysis the effect of pump magnetic field $\mathbf{B}_0$ is neglected by considering that ω_p and ω_c are comparable to ω_0.

The modulation process in the medium considered above must fulfil the phase matching conditions *i.e.* $\mathbf{k}_\pm = \mathbf{k}_0 \pm \mathbf{k}_a$ and $\omega_\pm = \omega_0 \pm \omega_a$ known as momentum and energy conservation relations, respectively. Here we have considered only the resonant side band frequencies $(\omega_0 \pm \omega_a)$ by assuming a long interaction path (by considering the crystal of infinite length) so that higher order scattering terms being off-resonant, are negligible and thus the only waves couple with the sound waves are the incident (ω_0) and scattered waves $(\omega_0 \pm \omega_a)$. On using equations (5–7), the perturbed electron density oscillating at the forced wave frequencies *i.e.* upper and lower side band frequencies may be obtained as

$$n_1(\omega_\pm, k_\pm) = \frac{\mathrm{i}k_a^3\beta^2 E_a\overline{\omega}_\mathrm{p}^2}{e\rho(\omega_a^2 + 2\mathrm{i}\omega_a\gamma_a - k_a^2v_a^2)} \times\left[\overline{\omega}_\mathrm{p}^2 - \omega_\pm^2 - \mathrm{i}\nu\omega_\pm + \mathrm{i}k_\pm\overline{E}_0\right]^{-1} \tag{8}$$

here v_a $(= \sqrt{c/\rho})$ is the velocity of acoustic wave in the medium. We have also assumed here that electron density perturbation at side band frequencies varies as $\exp\left[\mathrm{i}(k_\pm x - \omega_\pm t)\right]$. The density perturbations thus produced affect the propagation characteristics of the generated wave, which can be examined by employing the general electromagnetic wave equations.

With a view to study the contributions to the nonlinear current density due to the induced polarisation in the medium, the effect of the transition dipole moment has been neglected. Thus the resonant component of the induced nonlinear current densities due to density perturbation oscillating at the forced wave frequencies are

$$\mathbf{J}_+ = -en_1\left(\omega_+, \mathbf{k}_+\right)\mathbf{v}_0 \tag{9a}$$

$$\mathbf{J}_- = -en_1\left(\omega_-, \mathbf{k}_-\right)\mathbf{v}_0^* \tag{9b}$$

in which * denotes the complex conjugate of the respective term.

Henceforth treating induced polarisation $\mathbf{P}_\pm$ at the modulated frequencies as a time integral of the nonlinear current density $\mathbf{J}_\pm$, we have

$$\mathbf{P}_\pm = \int \mathbf{J}_\pm \mathrm{d}t. \tag{10}$$

The effective nonlinear polarisation due to modulated waves is obtained as

$$\mathbf{P}_\mathrm{e} = \mathbf{P}_+ + \mathbf{P}_-. \tag{11}$$

It is essential for the amplification of modulated waves that both the side bands should contribute equally and this modulation is then transferred to the acoustic mode which in turn gets amplified.

Thus from equations (8–11), we get the total effective polarisation as

$$|\mathbf{P}_\mathrm{e}| = \frac{\mathrm{i}k_a^3\beta^2\overline{\omega}_\mathrm{p}^2\left|\overline{E}_0\right||E_a|}{\omega_0\rho\left(\omega_a^2 + 2\mathrm{i}\omega_a\gamma_a - k_a^2v_a^2\right)} \times\left[\frac{1}{\omega_+}\left(\delta_1^2 - \mathrm{i}\nu\omega_+ + \mathrm{i}k_+\overline{E}_0\right)^{-1} - \frac{1}{\omega_-}(\delta_2^2 - \mathrm{i}\nu\omega_- + \mathrm{i}k_-E_0)^{-1}\right] \tag{12}$$

here $\delta_1^2 = \overline{\omega}_\mathrm{p}^2 - \omega_+^2,\ \delta_2^2 = \overline{\omega}_\mathrm{p}^2 - \omega_-^2$, and in deriving this equation we have used the plane wave approximation. It is obvious from equation (12) that in absence of piezoelectricity (*i.e.* $\beta = 0$) the total polarisation would become equal to zero. Hence in our study, consideration of piezoelectricity is essential.

On simplifying equation (12) and rearranging the various terms, the total nonlinear polarisation $\mathbf{P}_\mathrm{e}$ can be obtained as

$$\mathbf{P}_\mathrm{e} = \frac{2\varepsilon A e^2 \overline{\omega}_\mathrm{p}^2 k_a^2(\delta^2-\nu^2)}{m^2\omega_0^4(\omega_a^2-k_a^2v_a^2+2\mathrm{i}\omega_a\gamma_a)}\left|\mathbf{E}_\mathrm{e}\right|^2\mathbf{E}_a \times\left[\left\{(\delta^2+\nu^2)-\frac{2\nu k_a}{\omega_0}\overline{E}_0-\frac{(k_0^2-k_a^2)}{\omega_0^2}\overline{E}_0^2\right\}^2+\frac{4\delta^2k_a^2}{\omega_0^2}\overline{E}_0^2\right]^{-1} \quad (13)$$

in which $A=\kappa^2k_a^2v_a^2$, $\kappa^2=\beta^2/\varepsilon\rho$ and $\delta=\overline{\omega}_\mathrm{p}-\omega_0$.

The components of oscillatory fluid velocity $\mathbf{v}_0$ in the presence of pump $\mathbf{E}_0$ and static magnetic fields $\mathbf{B}_\mathrm{s}$ may be obtained from equation (2) as

$$v_{0x}=\frac{(e/m)E_{0x}+\omega_{\mathrm{c}z}v_{0y}}{(\nu-\mathrm{i}\omega_0)}, \quad (14a)$$

$$v_{0y}=\frac{\left[(e/m)[-\omega_{\mathrm{c}z}E_{0x}+(\nu-\mathrm{i}\omega_0)E_{0y}\right]}{(\omega_\mathrm{c}^2-\omega_0^2)}. \quad (14b)$$

Using Maclaurin's power series to expand equation (13), one obtains after some algebraic simplification, the following expression for total nonlinear polarisation:

$$\mathbf{P}_\mathrm{e} = \frac{2\varepsilon A e^2 \overline{\omega}_\mathrm{p}^2 k_a^2(\delta^2-\nu^2)}{m^2\omega_0^4(\omega_a^2-k_a^2v_a^2+2\mathrm{i}\omega_a\gamma_a)}\left|\mathbf{E}_\mathrm{e}\right|^2\mathbf{E}_a \times\left[(\delta^2+\nu^2)^{-2}+\frac{4\nu k_a}{\omega_0}(\delta^2+\nu^2)^{-3}\overline{E}_0 + \frac{2}{\omega_0^2}(5\nu^2k_a^2-3\delta^2k_a^2+2\delta^2k_0^2)(\delta^2+\nu^2)^{-4}\overline{E}_0^2+\cdots\right]. \quad (15)$$

2.1 Effective third-order susceptibility and steady-state gain coefficient

The third-order induced polarisation due to cubic nonlinearities at modulated frequencies ($\omega_\pm$) is defined as

$$\mathbf{P}_\mathrm{e}^{(3)}=\varepsilon_0\chi_\mathrm{e}^{(3)}\left|\mathbf{E}_\mathrm{e}\right|^2\mathbf{E}_a. \quad (16)$$

From equation (15) one may obtain the following expression for the third-order polarisation as

$$\mathbf{P}_\mathrm{e}^{(3)}=\frac{2\varepsilon A e^2 \overline{\omega}_\mathrm{p}^2 k_a^2(\delta^2-\nu^2)(\delta^2+\nu^2)^{-2}}{m^2\omega_0^4(\omega_a^2-k_a^2v_a^2+2\mathrm{i}\omega_a\gamma_a)}\left|\mathbf{E}_\mathrm{e}\right|^2\mathbf{E}_a. \quad (17)$$

Using equations (16, 17), the third-order susceptibility may be obtained as

$$\chi_\mathrm{e}^{(3)}=\frac{2\varepsilon_1 A e^2 \overline{\omega}_\mathrm{p}^2 k_a^2(\delta^2-\nu^2)(\delta^2+\nu^2)^{-2}}{m^2\omega_0^4(\omega_a^2-k_a^2v_a^2+2\mathrm{i}\omega_a\gamma_a)}\cdot \quad (18)$$

This equation reveals that the third-order susceptibility $\chi_\mathrm{e}^{(3)}$(*via* $\mathbf{P}_\mathrm{e}$) couples the perturbed density wave at side band frequencies $\omega_\pm$ and the acoustic wave at ω_a to produce modulational interaction.

It is also evident from equation (18) that $\chi_\mathrm{e}^{(3)}$ is a complex quantity and thus can be separated into real and imaginary parts as

$$\chi_\mathrm{e}^{(3)}=\chi_\mathrm{er}^{(3)}+\mathrm{i}\chi_\mathrm{ei}^{(3)} \quad (19)$$

where

$$\chi_\mathrm{er}^{(3)}=\frac{2\varepsilon_1 A e^2 \overline{\omega}_\mathrm{p}^2 k_a^2(\delta^2-\nu^2)(\omega_a^2-k_a^2v_a^2)}{m^2\omega_0^4(\delta^2+\nu^2)^2\left((\omega_a^2-k_a^2v_a^2)^2+4\omega_a^2\gamma_a^2\right)} \quad (20a)$$

and

$$\chi_\mathrm{ei}^{(3)}=\frac{-4\varepsilon_1 A e^2 \overline{\omega}_\mathrm{p}^2 k_a^2(\delta^2-\nu^2)\omega_a\gamma_a}{m^2\omega_0^4(\delta^2+\nu^2)^2\left((\omega_a^2-k_a^2v_a^2)^2+4\omega_a^2\gamma_a^2\right)} \quad (20b)$$

here the subscripts "r" and "i" to the susceptibility represent its real and imaginary parts, respectively. The above equations (20) describe the steady-state optical response of the medium in the presence of an oblique static magnetic field $\mathbf{B}_\mathrm{s}$ and govern the nonlinear wave propagation through the medium. It is evident that the nonlinear susceptibility is influenced by the unperturbed carrier concentration through ω_p and the external magnetic field $\mathbf{B}_\mathrm{s}$ through ω_c. Now the imaginary part (Eq. (20b)) of the third-order susceptibility can easily be employed to obtain steady-state growth and the real part (Eq. (20a)) is useful in exploring the dispersive characteristics of the medium for the propagating modulated wave.

From equation (20a), it may be inferred that $\chi_\mathrm{er}^{(3)}$ can have positive and negative values depending upon the relative values of ω_p and ν. Positive dispersion of the modulated wave occurs at $\omega_\mathrm{p}\gg\nu$ and under such circumstances the nonlinear refractive index falls off with distance from the beam axis. Snell's law allows us to conclude that beam velocity of the modulated wave increases with the distance from the beam axis. This leads to the occurrence of self-focusing of the signal. As $\chi_\mathrm{er}^{(3)}$ becomes more positive, one may expect more effective self-focusing of the modulated wave. However for nondispersive acoustic mode at $\omega_a\approx\mathbf{k}_a\cdot\mathbf{v}_a$, we observe anomalous dispersion of the modulated wave in the medium as $\chi_\mathrm{er}^{(3)}\to 0$ when $\omega_a\to\mathbf{k}_a\cdot\mathbf{v}_a$. It can further be concluded for nondispersive acoustic mode, there is no change in the refractive index of the medium due to induced current density.

In order to explore the possibility of modulational amplification in the semiconductor plasma, we employ the relation

$$\alpha_\mathrm{e}=\frac{k_a}{2\varepsilon_\mathrm{L}}\chi_\mathrm{ei}^{(3)}\left|\mathbf{E}_\mathrm{e}\right|^2 \quad (21)$$

here α_e is the effective nonlinear absorption coefficient. The nonlinear growth of the modulated signal is possible only if α_e obtained from equation (21) is negative. Thus from equations (20b, 21) it can be inferred that the growth g_e $(=|-\alpha_\mathrm{e}|)$ of the modulated wave can be achieved only when $\chi_{ei}^{(3)}$ is negative *i.e.* $\delta^2 > \nu^2$, which infers that the growth is possible only in highly doped semiconductors. This condition can be obtained by adjusting the doping level of the medium. Thus the growth of the modulated wave obtained from equations (20b, 21) as

$$g_e = \frac{-2\varepsilon_1 A e^2 \overline{\omega}_p^2 k_a^3 (\delta^2 - \nu^2)\omega_a \gamma_a}{\varepsilon_L m^2 \omega_0^4 (\delta^2 + \nu^2)^2 \left((\omega_a^2 - k_a^2 v_a^2)^2 + 4\omega_a^2 \gamma_a^2\right)} |\mathbf{E}_e|^2 \tag{22}$$

or the steady-state gain of the modulated wave in n-InSb crystal (data are given in the Sect. 3) may be obtained as

$$g_e = 1.385 \times 10^{-4} I_{in} \tag{23}$$

where we have defined $I_{in} = (1/2)\eta\varepsilon_0 c_0 |\mathbf{E}_e|^2$ in which c_0 is the speed of light in vacuum and η being the background refractive index of the crystal.

2.2 Transient gain factor

It is evident from equation (23) that a high power pulsed laser can only yield a significant growth rate of the modulated signal but with high power pump the study of transient effects becomes unavoidable. The advantage of incorporating the transient effects in our analysis is that we can also predict threshold pump intensity (I_{th}) for the onset of modulation process with optimum pulse duration for the modulational instability to occur. In general, the transient gain factor g_T is related to steady-state gain factor g_e through the relation [27]

$$g_T = (2g_e x \Gamma \tau_p)^{1/2} - \Gamma\tau_p \tag{24}$$

where Γ being the optical photon life time, x is the interaction length and τ_p is the pulse duration. For very short pulse ($\tau_p \le 10^{-10}$ s), the interaction length should be replaced by $(c_l\tau_p/2)$, where c_l is the speed of light in the crystal and is given by $(c_0/\sqrt{\varepsilon_L})$ where ε_L is the lattice dielectric constant of the material. By making $g_T = 0$, in equation (24) we can obtain the threshold pump intensity as

$$I_{th} = \frac{\Gamma}{2G_e c_l} \tag{25}$$

with $G_e = g_e/I_{in}$, the steady-state gain per unit pump intensity.

Numerical analysis have been done using $\Gamma = 4 \times 10^8$ s^{-1} for n-InSb crystal, which yields the threshold value of the pump intensity for the onset of modulational instability as 2.253×10^9 Wm^{-1}.

However for $\tau_p \ge 10^{-9}$ s, the cell length can be taken equal to "x", under such conditions we find

$$g_T = (\Gamma\tau_p)^{1/2}\left[-(\Gamma\tau_p)^{1/2} + (g_e x)^{1/2}\right]\cdot \tag{26}$$

Using above expression we get the idea about the optimum pulse duration $(\tau_p)_{opt}$ above which no gain would be achieved. By making $g_T = 0$, equation (26) yields

$$(\tau_p)_{opt} \approx \frac{g_e x}{\Gamma}\,. \tag{27}$$

A calculation for n-InSb using the values of g_e obtained earlier and $x = 10^{-4}$ m gives

$$(\tau_p)_{opt} = (4.064 \times 10^{-11} I_{in})\ \text{s}. \tag{28}$$

This value of $(\tau_p)_{opt}$ not only explains the washing out of the gain of modulated wave at large pulse duration but also suggests that optimum pulse duration can be increased by increasing the intensity of the pump.

3 Results and discussions

This section is devoted to the detailed numerical study of the modulational instability of a semiconductor crystal arising due to third-order susceptibility of the medium. The consequent amplification of modulated waves resulting due to transfer of modulation from the pump wave to the signal wave and resulting gain factors have been dealt with in the present section.

The analytical results obtained are applied to a piezoelectric semiconductor like n-InSb at 77 K irradiated by a pulse 10.6 μm CO_2 laser. The following set of parameters has been used in the analysis: $m = 0.014m_0$, $\beta = 0.054$ C m^{-2}, $\rho = 5.8 \times 10^3$ kg m^{-3}, $\varepsilon_L = 18.0$, $\varepsilon_1 = 15.8$, $\nu = 4 \times 10^{11}$ s^{-1}, $n_e = 10^{24}$ m^{-3}, where m_0 being the free electron mass.

A detailed investigation about the nature of the steady-state gain factor reveals that an appreciable amplification of modulated wave ($g_e \approx 10^4$–10^6 m^{-1}) is obtainable only when $\omega_a \approx \mathbf{k}_a \cdot \mathbf{v}_a$ (*i.e.* condition of anomalous dispersion). Growth rate of the modulated wave is found to be independent of its frequency instead it depends on the frequency of the pump and acoustic wave; a fact in agreement with experimental observations [28].

The expression for the steady-state growth rate as obtained from equation (22) has the usual

$$g_e \propto \left[ak_a^2(b|E_e|^2 - ak_a^2)\right]^{1/2}$$

nature as predicted by Drake *et al.* [29]. The gain constant g_e has the usual characteristic dependence on the wave vector. It can be inferred from equation (22) that for lower magnitude of k_a, (such that $\omega_a \gg k_a v_a$) g_e increases with k_a and at $\omega_a \approx k_a v_a$, *i.e.* for nondispersive acoustic mode g_e is maximum and further when $\omega_a < k_a v_a$, g_e shows a steep decline with increasing k_a. This nature of variation of the steady-state growth rate with k_a is in confirmation with the usual dependence quoted above.

The numerical estimations (using Eq. (22)) dealing with the carrier density influencing the steady-state growth rate g_e $(= |-\alpha_e|)$ are plotted in Figure 1 for a nondispersive acoustic mode *i.e.* at $\omega_a \approx \mathbf{k}_a \cdot \mathbf{v}_a$ (with $k_a = 2.08 \times 10^8$ m^{-1}, $v_a = 4 \times 10^3$ m s^{-1} and $\omega_a = 8.32 \times 10^{11}$ s^{-1}). Figure 1 shows the dependence of g_e on the carrier density n_e through the plasma frequency ω_p when $\omega_c = 0.9\omega_0$, $\theta = 45°$, $E_e = 10^7$ V m^{-1}. We have considered the range of plasma frequency from 2.4×10^{14} to 2.7×10^{14} s^{-1}, the corresponding carrier density would be from 4.57×10^{24} to 5.78×10^{24} m^{-3}. This range of free

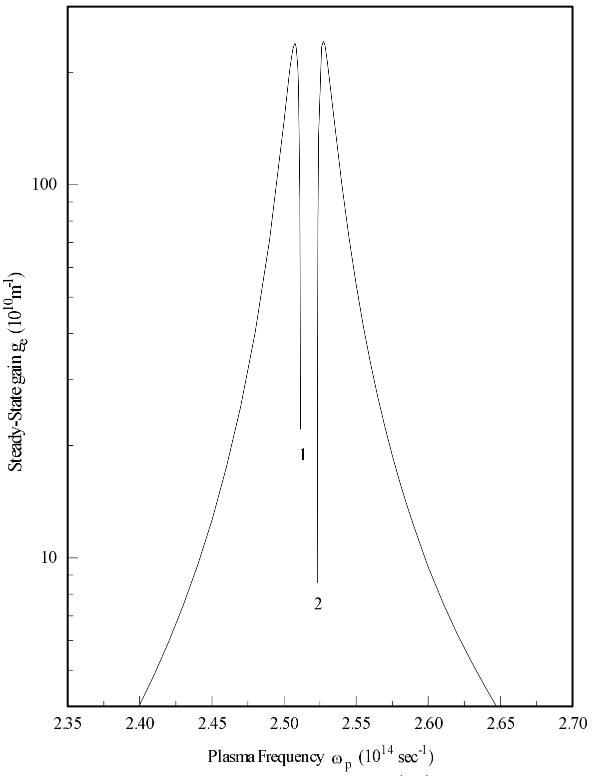

Fig. 1. Variation of steady-state gain (g_{e}) with plasma frequency (ω_{p}) when $\omega_{\mathrm{c}} = 0.9\omega_0$, $\theta = 45°$, $E_{\mathrm{e}} = 10^7$ V m^{-1}. 1: 2.5115×10^{14} s^{-1} and 2: 2.5230×10^{14} s^{-1}.

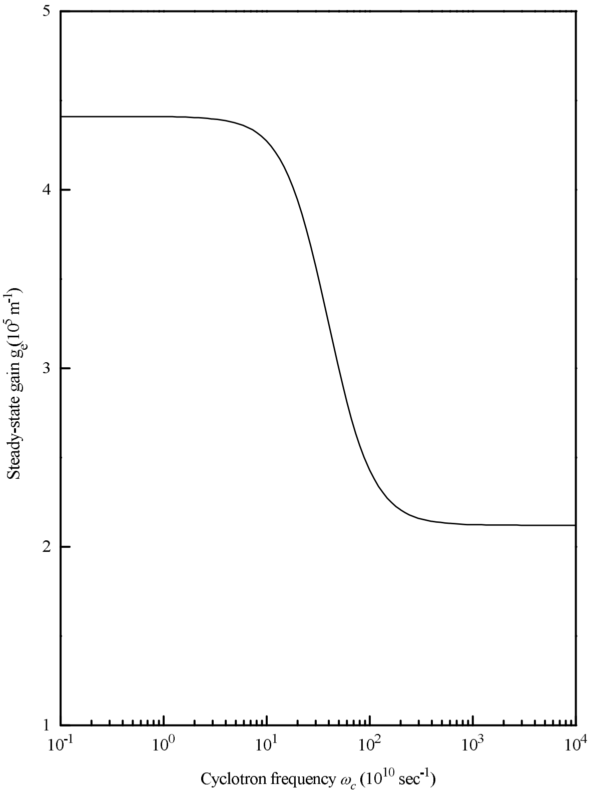

Fig. 2. Variation of steady-state gain (g_{e}) with cyclotron frequency (ω_{c}) when $\omega_{\mathrm{p}} = 1.12 \times 10^{14}$ s^{-1}, $\theta = 45°, E_{\mathrm{e}} = 10^7$ V m^{-1}.

carrier density may easily be obtained by varying doping level. But it is also true that one may work with a crystal of fixed density at a time, thus this variation is not so important for discussion. Nevertheless, through this variation one may choose the most appropriate density level for a particular use. It is found from Figure 1 that the steady-state gain first increases until the plasma frequency reaches 2.51×10^{14} s^{-1}, and then suddenly decreases very sharply and becomes minimum at $\omega_{\mathrm{p}} \approx 2.5115 \times 10^{14}$ s^{-1}. In the region $2.5115 \times 10^{14} < \omega_{\mathrm{p}} < 2.5230 \times 10^{14}$ s^{-1}, one does not get amplification of the wave but attenuation as in this region $\delta^2 < \nu^2$. This region is illustrated by discontinuity of the curve in the plot. For $\omega_{\mathrm{p}} > 2.5230 \times 10^{14}$ s^{-1} the gain constant again starts increasing very sharply and achieve a maximum value, after which it once again decreases. Thus the presence of an external magnetic field for which $\omega_{\mathrm{c}} = 0.9\omega_0$, one can get considerably large amount of gains at $\omega_{\mathrm{p}} = 2.5075 \times 10^{14}$ and 2.5271×10^{14} s^{-1}. The corresponding carrier densities would be 4.985×10^{24} and 5.063×10^{24} m^{-3}. Thus crystals with these values of carrier concentration yield maximum steady-state gain for the said interaction. The above quoted variation of steady-state gain constant with plasma frequency may very easily be utilised in construction of optical switches. A variation (using Eq. (22)) of steady-state gain g_{e} with magnetostatic field $\mathbf{B}_{\mathrm{s}}$ *via* cyclotron frequency ω_{c} is shown in Figure 2. One may infer from the figure that g_{e} remains almost constant for lower values of ω_{c} (upto 10^{11} s^{-1}). Thereafter it starts decreasing as ω_{c} approaches ν. This trend continues until $\omega_{\mathrm{c}} \approx \overline{\omega}_{\mathrm{p}}$. A further increase in ω_{c} (in the region $\omega_{\mathrm{c}} \geq \overline{\omega}_{\mathrm{p}}$) stabilises g_{e} once again and it becomes independent of ω_{c}. We have plotted the variation in the range $10^9 < \omega_{\mathrm{c}} < 1.7 \times 10^{14}$ s^{-1} for which corresponding strength of magnetic field would be $7.96 \times 10^{-5} < |\mathbf{B}_{\mathrm{s}}| < 11$ T, which can very easily be obtained by using electromagnets.

We now address ourselves to the question of the behaviour of the transient gain g_{T} as a function of the pump pulse duration τ_{p}. For this purpose, we have used equation (26) and considered pulses with duration in the range $10^{-10} \leq \tau_{\mathrm{p}} \leq 10^{-8}$ s. The cell length is taken as 10^{-4} m. Figure 3 represents the qualitative behaviour of the transient gain factor g_{T} of modulated signal as a function of the pulse duration with pump intensity I_{in} as a parameter. g_{T} first increases with τ_{p} when I_{in} is fixed, and attains maximum value. Curves I, II and III show that the rise in I_{in} shifts the maximum gain point towards higher value of τ_{p}. Keeping I_{in} fixed, if τ_{p} is chosen beyond the maximum gain point, g_{T} diminishes very rapidly.

It has also been found that heavily doped semiconductors are the most appropriate host for modulation processes. Hence larger amplification of the waves can be attained by increasing the carrier concentration of the magnetised medium by n-type doping in the crystal. However doping should not exceed the limit for which the effective plasma frequency $\overline{\omega}_{\mathrm{p}}$ exceeds the input pump frequency ω_0. Because in the regime when $\overline{\omega}_{\mathrm{p}} > \omega_0$, the electromagnetic pump wave will be reflected back by the intervening medium.

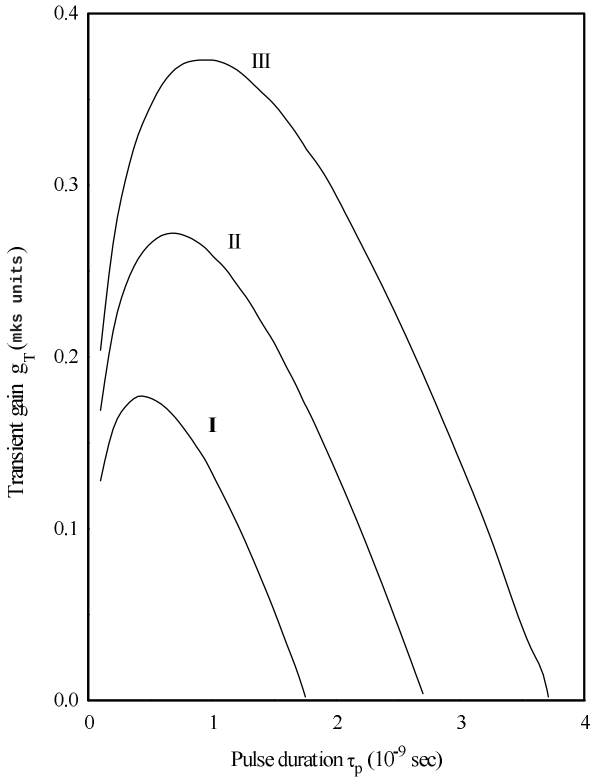

Fig. 3. Variation of transient gain ($g_{\rm T}$) with the pump pulse duration ($\tau_{\rm p}$) when $\omega_{\rm c} = 0.9\omega_0$, $\theta = 90°$ with the pump intensity ($I_{\rm in}$) as a parameter. Curves I, II and III correspond to $I_{\rm in} = 7.586 \times 10^{10}$, 1.165×10^{11}, and 1.60×10^{11} W m^{-2}, respectively.

One can also estimate the approximate value of the third-order optical susceptibility $\chi_{\rm e}^{(3)}$ of the crystal in the present scheme. It may be recalled that we have completely neglected the influence of the virtual transitions and assumed the induced nonlinear current density as the origin of induced polarisation. Equation (18) under appropriate numerical assumptions can be used to find

$$\chi_{\rm e}^{(3)} \approx \frac{\omega_{\rm p}^2 e^2 \varepsilon_1 \kappa^2 k_a^4 v_a (\omega_{\rm c\mathit{x}}^2 + \nu^2)}{m^2 \omega_a \Gamma_a^2 \delta^2 (\omega_{\rm c}^2 + \nu^2)} \cdot \qquad (29)$$

We find form equation (29) that crystals with $n_{\rm e} = 10^{24}$ m^{-3}, $\chi_{\rm e} \approx 10^{-7}$ esu and $n_{\rm e} = 10^{22}$ m^{-3}, $\chi_{\rm e} \approx 10^{-11}$ esu whereas it has been experimentally observed that the third-order optical susceptibility for III-V semiconductor is of the order of 10^{-12} esu with carrier concentration of 10^{22} m^{-3} [24]. Thus one may infer that the doping concentration plays an important role in raising the third-order optical susceptibility of the crystal. The magnetic field (in terms of $\omega_{\rm c}$) also effectively raises the third-order optical susceptibility of the medium. By raising the magnitude of $\chi_{\rm e}^{(3)}$, it is possible to incite third-order nonlinear phenomena at much lower pump amplitudes. Thus in III-V crystals, modulational interaction in the infrared regime appears quite promising under the typical resonance condition *viz.* $\overline{\omega}_{\rm p} \approx \omega_0$ and replaces the conventional idea of using high power lasers.

From the above discussions it may be concluded that the magnitude and orientation of applied external magnetic field are favourable for the onset of the modulational amplification of the modulated wave in heavily doped regimes. The highlight of our study is the modulation process (steady-state as well as transient) can be studied in noncentrosymmetric dielectric medium only with the knowledge of piezoelectric potential, using a simple hydrodynamic treatment.

One of us (GS) is thankful to University Grants Commission, New Delhi for the award of Teacher Research Fellowship to him.

References

1. A.A. Galeev, R.Z. Sagdeev, Nucl. Fission **13**, 603 (1973).
2. K.A. Brueckner, S. Jorna, Rev. Mod. Phys. **46**, 325 (1974).
3. G.P. Agarwal, in *Nonlinear Fiber Optics* (Academic Press, Boston, 1984), Sect. 5.
4. S.A. Akhmanov, V.A. Vyslolukh, A.S. Chirkin, Phys. Usp. **29**, 642 (1986).
5. V.I. Bespalov, V.I. Talonov, JETP Lett. **3**, 307 (1966).
6. R. Muller, Phys. Stat. Sol. b **150**, 587 (1988).
7. S. Trillo, S. Wabnitz, G.I. Stegeman, E.M. Bright, J. Opt. Soc. Am. **6**, 889 (1989).
8. C.F. Quate, R.B. Thompson, Appl. Phys. Lett. **16**, 299 (1970).
9. R.B. Thompson, C.F. Quate, J. Appl. Phys. **42**, 907 (1971).
10. K.L.I. Kobayashi, E. Otsuka, *XI International Conf. Phys.* Semiconductors **1**, 903 (1972).
11. A. Sen, P.K. Kaw, J. Phys. D **6**, 2097 (1973).
12. M.C. Steele, B. Vural, in *Wave Interaction in Solid State Plasmas* (McGraw Hill, New York, 1969), pp. 137-158.
13. S. Ghosh, V.K. Agrawal, Ind. J. Phys. A **59**, 55 (1985).
14. S. Ghosh, V.K. Agrawal, Phys. Stat. Sol. b **102**, K107 (1980).
15. E.M. Conwell, A.K. Ganguli, Phys. Rev. B **4**, 2535 (1971).
16. H.N. Spector, Phys. Rev. B **7**, 1420 (1973).
17. Md. Salimullah, Tarsen Singh, J. Phys. Chem. Sol. **43**, 1087 (1982).
18. S. Ghosh, D.K. Sinha, Acoustica **58**, 298 (1985).
19. C.N. Lashmore-Davies, Phys. Fluids **19**, 587 (1976).
20. D.R. Anderson, S, Datta, R.L. Gunshor, J. Appl. Phys. **54**, 5608 (1983).
21. S. Wabnitz, Phys. Rev. A **38**, 2018 (1988).
22. G.P. Agrawal, P.L. Baldeck, R.R. Alfano, Phys. Rev. A **39**, 3406 (1989).
23. N. Bloembergen, in *Quantum Optics*, edited by S.M. Kay, A. Mait Land (Academic Press, London, 1970), pp. 387-392.
24. N. Bloembergen, in *Nonlinear Optics* (Benjamin, New York, 1965).
25. P. Gadkari, S. Ghosh, Phys. Stat. Sol. b **172**, 709 (1992).
26. S. Guha, S. Ghosh, P.K. Sen, Phys. Stat. Sol. **52**, 407 (1979).
27. C.S. Wang, in *Quantum Electronics*, edited by H. Rabin, C.L. Tang (Academic Press, New York, 1975), Vol. 1, part A, pp. 447-472.
28. A. Yariv, *Optical Electronics in Modern Communications* (Academic Press, New York, 1997).
29. J.F. Drake, P.K. Kaw, C.Y. Lee, G. Schmidt, C.S. Liu, M.N. Rosenbluth, Phys. Fluids **17**, 778 (1974).